



Funciones y sus propiedades

Viernes 08 de Enero de 2021

Ayudantía 08

1. Determine la composición $f \circ g$ (en ese orden) de las siguientes funciones. Indique dominio, regla de asignación y recorrido (en ese orden):

a. $g: \{1, 2, 3, 4\} \rightarrow \mathbb{Z}, g(x) = 2x + 1$
 $f: \{a \in \mathbb{Z} : 1 \leq a \leq 15\} \rightarrow \mathbb{Z}, f(x) = 1 - 3x$

b. $g: \mathbb{Z} \rightarrow \mathbb{Z}, g(x) = 2x + 1$
 $f: \{a \in \mathbb{Z} : 1 \leq a \leq 15\} \rightarrow \mathbb{Z}, f(x) = 1 - 3x$

c. $g: \left\{ a \in \mathbb{Q} : \frac{1}{2} < a < \frac{10}{3} \right\} \rightarrow \mathbb{Q}, g(x) = 2 + \frac{1}{x}$
 $f: \{x \in \mathbb{Q} : 2 \leq x \leq 5\} \rightarrow \mathbb{Q}, f(x) = 1 + \frac{3}{x}$

1a. Composición f o g

	x	f	$f(x)$		x	$f \circ g$	$f \circ g(x)$		x	$f \circ g$	$f \circ g(x)$
	1	\rightarrow	$1-3(1)=-2$		1	\rightarrow	-8		1	\rightarrow	$-2-6(1)=-8$
	2	\rightarrow	$1-3(2)=-5$		2	\rightarrow	-14		2	\rightarrow	$-2-6(2)=-14$
x	g	$g(x)$	3	\rightarrow	$1-3(3)=-8$		3	\rightarrow	3	\rightarrow	$-2-6(3)=-20$
1	\rightarrow	$2 \cdot (1)+1=3$	4	\rightarrow	$1-3(4)=-11$		4	\rightarrow	4	\rightarrow	$-2-6(4)=-26$
2	\rightarrow	$2 \cdot (2)+1=5$	5	\rightarrow	$1-3(5)=-14$						
3	\rightarrow	$2 \cdot (3)+1=7$	6	\rightarrow	$1-3(6)=-17$						
4	\rightarrow	$2 \cdot (4)+1=9$	7	\rightarrow	$1-3(7)=-20$						
			8	\rightarrow	$1-3(8)=-23$						
			9	\rightarrow	$1-3(9)=-26$						
			\vdots								

$\text{Dom}(f \circ g) = \{1, 2, 3, 4\} = \text{Dom}(g)$ $\text{Rec}(f \circ g) = \{-8, -14, -20, -26\}$ subconjunto de $\text{Rec}(f)$

Regla de Asignación: $g(x) = 2x+1$ $f(y) = 1 - 3y$

$$f \circ g(x) = f [g(x)] = f [2x+1] = 1 - 3[2x+1] = 1 - 6x - 3 = -2 - 6x$$

$f(x) = x^2$ $\text{Dom}(f) = \mathbb{R}$ $\text{Dom}(f) = [0, 9]$

1b. Composición f o g

x	g	$g(x)$	x	f	$f(x)$
	\vdots		1	\rightarrow	$1-3(1)=-2$
-1	\rightarrow	$2 \cdot (-1)+1=-1$	2	\rightarrow	$1-3(2)=-5$
0	\rightarrow	$2 \cdot (0)+1=1$	3	\rightarrow	$1-3(3)=-8$
1	\rightarrow	$2 \cdot (1)+1=3$	4	\rightarrow	$1-3(4)=-11$
2	\rightarrow	$2 \cdot (2)+1=5$	5	\rightarrow	$1-3(5)=-14$
3	\rightarrow	$2 \cdot (3)+1=7$	6	\rightarrow	$1-3(6)=-17$
4	\rightarrow	$2 \cdot (4)+1=9$	7	\rightarrow	$1-3(7)=-20$
5	\rightarrow	$2 \cdot (5)+1=11$	8	\rightarrow	$1-3(8)=-23$
6	\rightarrow	$2 \cdot (6)+1=13$	9	\rightarrow	$1-3(9)=-26$
7	\rightarrow	$2 \cdot (7)+1=15$	10	\rightarrow	$1-3(10)=-29$
8	\rightarrow	$2 \cdot (8)+1=17$	11	\rightarrow	$1-3(11)=-32$
	\vdots		12	\rightarrow	$1-3(12)=-35$
			13	\rightarrow	$1-3(13)=-38$
			14	\rightarrow	$1-3(14)=-41$
			15	\rightarrow	$1-3(15)=-44$

Dom (f o g) = {0,1,2,3,4,5,6,7} subconjunto de Dom(g)

Rec(f o g) = {-2,-8,-14,-20,-26,-32,-38,-44} subconjunto de Rec(f)

Regla de Asignación: $g(x) = 2x+1$ $f(y) = 1 - 3y$

$$f \circ g(x) = f [g(x)] = f [2x+1] = 1 - 3[2x+1] = 1 - 6x - 3 = -2 - 6x$$

1c.

$$c. \quad g: \left\{ a \in \mathbb{Q} : \frac{1}{2} < a < \frac{10}{3} \right\} \longrightarrow \mathbb{Q}, \quad g(x) = 2 + \frac{1}{x}$$

$$f: \{x \in \mathbb{Q} : 2 \leq x \leq 5\} \longrightarrow \mathbb{Q}, \quad f(x) = 1 + \frac{3}{x}$$

Dom (f o g) = {x en Dom(g) : g(x) en Dom(f)} = Dom(g) , pues:

$$\frac{1}{2} < x < \frac{10}{3}$$

$$\text{Dom (g) : } \Rightarrow 2 > \frac{1}{x} > \frac{3}{10}$$

$$x > 1 \implies 1/x < 1 \quad x > 3/2 \implies 1/x < 2/3$$

$$\Rightarrow 4 > 2 + \frac{1}{x} > \frac{23}{10} = 2,3$$

$$x < a/b \implies 1/x > b/a$$

$$\text{Regla de asignación: } f \circ g(x) = f[g(x)] = 1 + \frac{3}{[g(x)]} = 1 + \frac{3}{\left[2 + \frac{1}{x}\right]} = \frac{2 + \frac{1}{x} + 3}{2 + \frac{1}{x}} = \frac{\frac{5x+1}{x}}{\frac{2x+1}{x}} = \frac{5x+1}{2x+1}$$

$$\frac{1}{2} < x < \frac{10}{3}$$

$$\Rightarrow \frac{3}{10} < \frac{1}{x} < 2$$

$$\Rightarrow \frac{23}{10} < 2 + \frac{1}{x} < 4$$

$$\Rightarrow \frac{23}{10} < \frac{2x+1}{x} < 4$$

$$\text{Recorrido: } \Rightarrow \frac{23}{30} < \frac{2x+1}{3x} < \frac{4}{3}$$

Por lo tanto, Rec(f o g) = { a en Q : 7/4 < a < 53/23 }

$$\Rightarrow \frac{3}{4} < \frac{3x}{2x+1} < \frac{30}{23}$$

$$\Rightarrow \frac{7}{4} < \frac{3x}{2x+1} + 1 < \frac{53}{23}$$

$$\Rightarrow \frac{7}{4} < \frac{3x+2x+1}{2x+1} < \frac{53}{23}$$

$$\Rightarrow \frac{7}{4} < \frac{5x+1}{2x+1} < \frac{53}{23}$$