22202 Álgebra I - Ayudantía 04

Más sobre Conjuntos

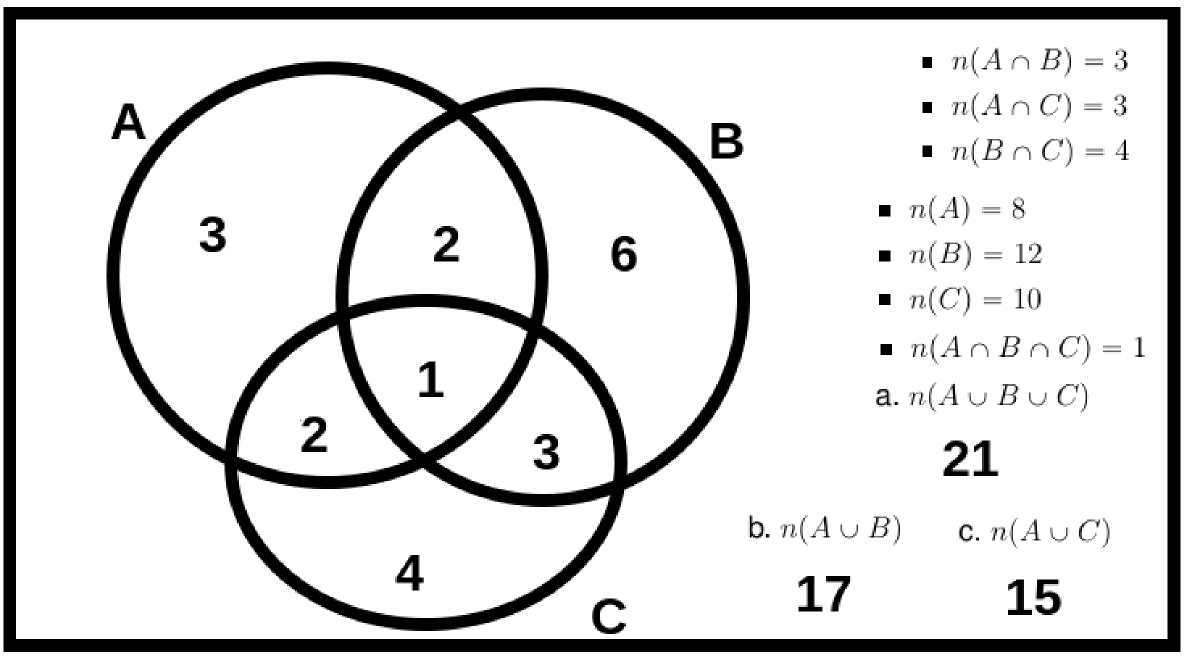
Viernes 27 de Noviembre de 2020

1. Se tienen tres conjuntos A, B, C tales que:

- $n(A \cap B) = 3$
- $n(A \cap C) = 3$
- $n(B \cap C) = 4$
- $n(A) = 8$
- $n(B) = 12$
- $n(C) = 10$
- $n(A \cap B \cap C) = 1$

Determine: a. $n(A \cup B \cup C)$ b. $n(A \cup B)$ c. $n(A \cup C)$

Solución: Usando Diagrama de Venn, se obtiene:



a.

$$\begin{aligned} n(A\cup B\cup C) &= n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C) \\ &= 8+12+10-3-3-4+1 \\ &= 21 \end{aligned}$$

b.

$$\begin{aligned} n(A\cup B) &= n(A)+n(B)-n(A\cap B) \\ &= 8+12-3 \\ &= 17 \end{aligned}$$

c.

$$\begin{aligned} n(A\cup C) &= n(A)+n(C)-n(A\cap C) \\ &= 8+10-3 \\ &= 15 \end{aligned}$$

3. Sean A, B, C conjuntos cualesquiera. Demostrar que:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Solución:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\begin{aligned}
\Rightarrow \quad n(A \cup B \cup C) &= n[(A \cup B) \cup C] \\
&= n(A \cup B) + n(C) - n[(A \cup B) \cap C] \\
&= n(A) + n(B) - n(A \cap B) + n(C) - n[(A \cup B) \cap C] \\
&= n(A) + n(B) + n(C) - n(A \cap B) - n[(A \cup B) \cap C] \\
&= n(A) + n(B) + n(C) - n(A \cap B) - n[(A \cap C) \cup (B \cap C)] \\
&= n(A) + n(B) + n(C) - n(A \cap B) - [n(A \cap C) + n(B \cap C) - n[(A \cap C) \cap (B \cap C)]] \\
&= n(A) + n(B) + n(C) - n(A \cap B) - [n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)] \\
&= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)
\end{aligned}$$

¿

Por lo tanto, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

4. Se define la ***diferencia simétrica*** entre dos conjuntos A y B como el conjunto:

$$\boxed{A \Delta B = (A - B) \cup (B - A)}$$

Demostrar las siguientes afirmaciones:

a. $A \Delta B = (A \cup B) - (A \cap B)$

b. $A \Delta A = \varnothing$

c. $A \Delta \varnothing = A$

d. $A \Delta B = B \Delta A$

e. $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

f. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

Dem: 4 a.

$$\begin{aligned}
A \Delta B &= (A - B) \cup (B - A) && \textit{Definición de resta} \\
&= (A \cap B^c) \cup (B \cap A^c) && \textit{Distributividad de } \cup \\
&= [(A \cap B^c) \cup B] \cap [(A \cap B^c) \cup A^c] && \textit{Conmutatividad de } \cup \\
&= [B \cup (A \cap B^c)] \cap [A^c \cup (A \cap B^c)] && \textit{Distributividad de } \cup \\
&= [(B \cup A) \cap (B \cup B^c)] \cap [(A^c \cup A) \cap (A^c \cup B^c)] && P \cup P^c = U \\
&= [(B \cup A) \cap U] \cap [U \cap (A^c \cup B^c)] && P \cap U = P \\
&= [B \cup A] \cap [A^c \cup B^c] && (A \cap B)^c = A^c \cup B^c \\
&= [A \cup B] \cap [A \cap B]^c && \textit{Definición de resta} \\
&= (A U B) - (A \cap B)
\end{aligned}$$

Por lo tanto, $A \Delta B = (A \cup B) - (A \cap B)$

